EDS 212: Day 1, Lecture 2

Functions, graphs, and linear functions

August 3rd, 2026

Functions

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For example, $y = 3x - 5$ is a function that tells us the **value of y** at **any value of x**. In this scenario, we would probably say **y is a function of x**.

$$x = \frac{y+5}{3}$$

Could you also rewrite it and say **x is a function of y**?

Here, with no knowledge of what's an input and what's an output, sure - but usually in environmental data science we specify the input variable(s), and the output variable(s) carefully.

What follows is the expression in the format of:

"**[output variable(s)] is/are a function of [input variable(s)]**".

Also: output variable = **dependent variable** and input variable = **independent variable**.

Thinking about inputs and outputs

For the following combinations of related variables, which do you expect would be the **input** and the **output** in a function describing how they are related? E.g. "Evapotranspiration is a function of air temperature."

✏️ Write your answer individually as **a sentence**.

1. fuel (biomass) / slope / wildfire severity / windspeed / air temperature
2. wind speed / power generated by wind turbine
3. soil C:N ratio / bacterial biomass / soil water content / leaf litter decomposition rate

Function notation

Single variable:

$$f(x) = \text{expression containing } x$$

Multivariable function (several independent variables)

$$g(a, T, z) = \text{expression containing } a, T \text{ and } z.$$

Function notation

Single variable (univariate) function:

$f(x) = [\textit{expression containing } x]$

Multivariate function:

$g(a, T, z) = [\textit{expression containing } a, T, \textit{ and } z]$

Evaluating functions

We evaluate functions by plugging in values of the input variables.

✏️ Example:

Evaluate $g(x, t) = 2.4x + 0.5t^2$ at $x = 3$ and $t = 10$

$$g(3, 10) = 2.4(3) + 0.5(10)^2$$

$$\boxed{= 57.2.}$$

Evaluating functions

We evaluate functions by plugging in values of the input variables.

✏️ Example:

Evaluate $g(x, t) = 2.4x + 0.5t^2$ at $x = 3$ and $t = 10$

$g(3, 10) = 2.4(3) + 0.5(10^2) = 57.2$

Graphs

* move to website. Come back to notes after break.

Why graphs?

Graphs are a way for us to more easily process trends or patterns that may be more challenging to understand in a table or list.

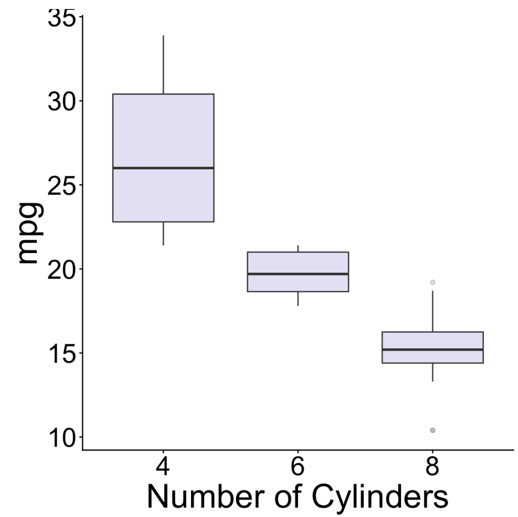
Which looks better and is easier to understand?

Show 4 entries Search:

	mpg	cyl	disp	hp
Mazda RX4	21	6	160	110
Mazda RX4 Wag	21	6	160	110
Datsun 710	22.8	4	108	93
Hornet 4 Drive	21.4	6	258	110

Showing 1 to 4 of 32 entries

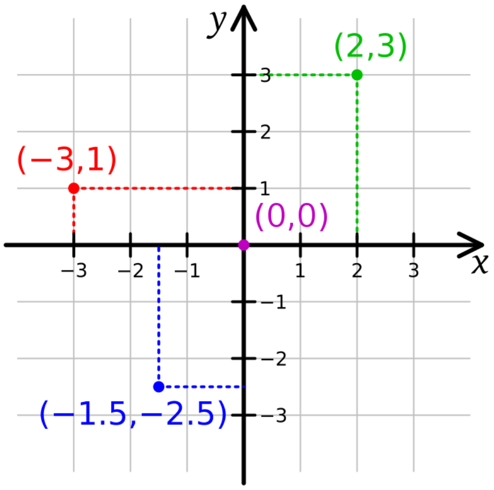
Previous 1 2 3 4 5 … 8 Next



The xy coordinate system

Graphs generally move in a 2-dimensional plane with a coordinate system using two axes:

- x-axis (horizontal axis)
- y-axis (vertical axis)



Axes units must be defined depending on the application.

We use pairs of numbers to place data:

- A point on the xy plane with **coordinates** (a, b) means the point is located at a on the x-axis and at b on the y-axis.
- Where would (2,-1) go on the graph?

We can join series of points to make graphs

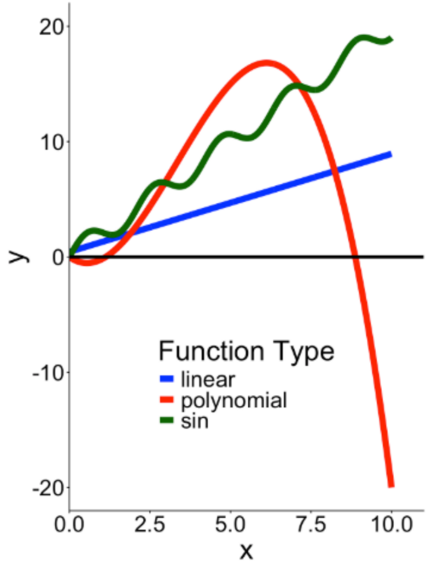
Functions on a single variable and a single output can be graphed

Two key ingredients to graphs

Intercepts

- x-intercept
  - Where does the graph intersect the x-axis?
  - In other words, for what values of x do we have $(x, 0)$ be part of the graph?

What are the intercepts of the polynomial function in red?

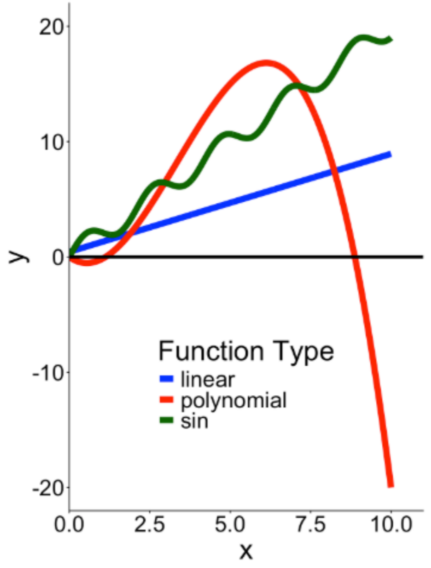


Two key ingredients to graphs

Intercepts

- x-intercept
  - Where does the graph intersect the x-axis?
  - In other words, for what values of x do we have $(x, 0)$ be part of the graph?
- y-intercept
  - Where does the graph intersect the y-axis?
  - In other words, for what value of y do we have $(0, y)$ be part of the graph?

What are the intercepts of the polynomial function in red?



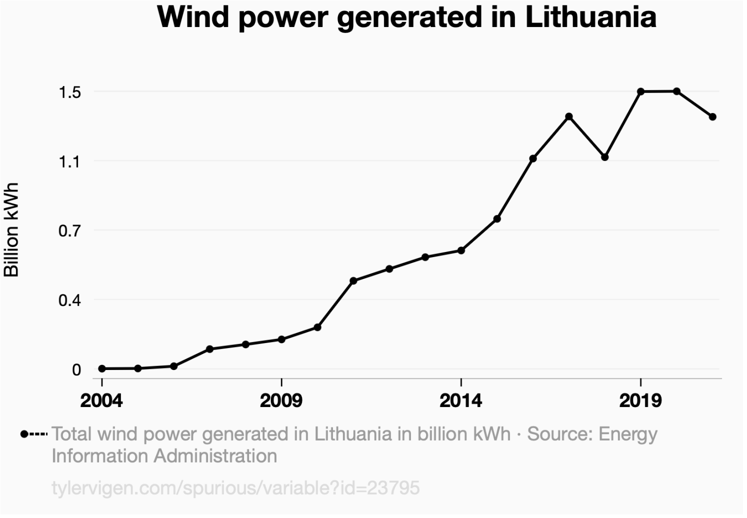
Understanding graphs

When you look at graphs, the first things you should ask:

- What variables are being plotted (e.g. x- & y-axis, including units)?
- What values are plotted (e.g. raw values, transformed, means, etc.)?
- What are the overall takeaways and am I understanding them responsibly?

Reading graphs (in general)

> "On the x-axis we have [x-variable] measured in [units] and on the y-axis the the [y-variable] measured in [units]. This figure shows the [change/pattern/relationship] between [x-variable] and [y-variable]. Overall [overall statement of pattern / trend / findings]."



Reading graphs (in general)

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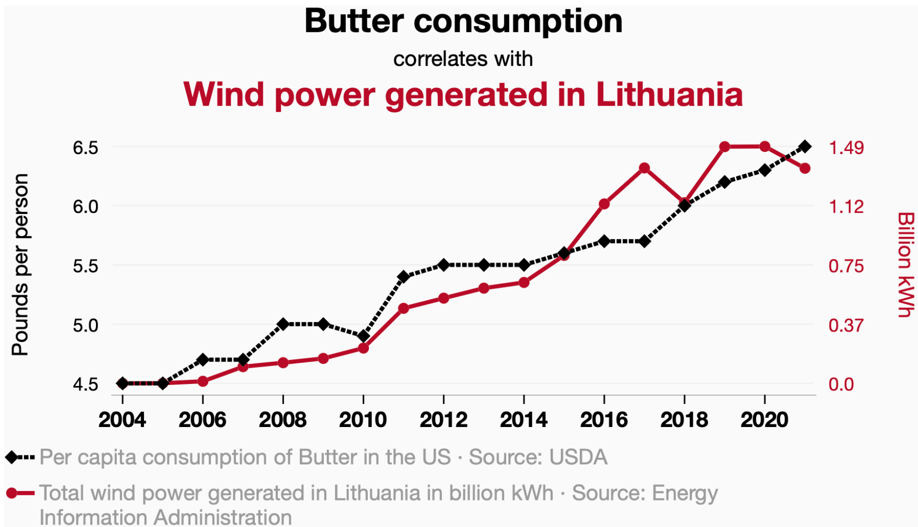
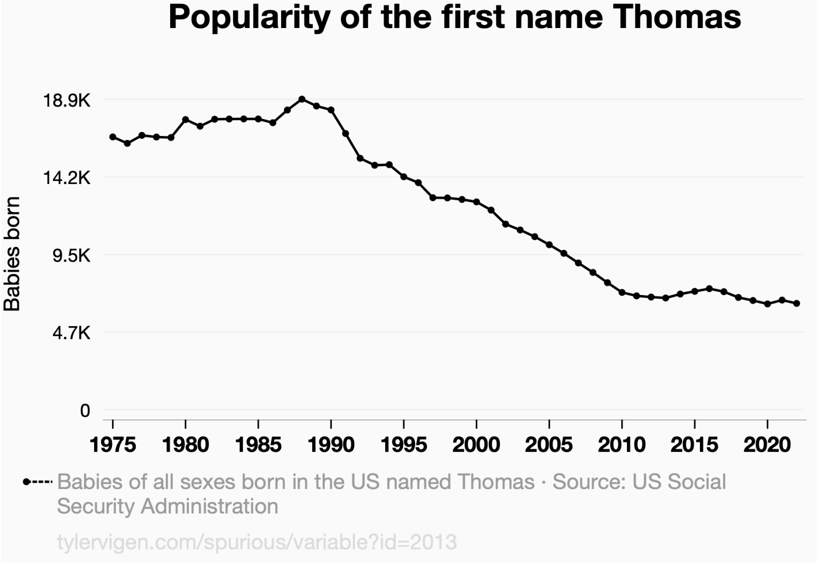


Source: TylerVigen.com

Reading graphs practice

💬 Partner 1 gets to read the graph for the first variable.

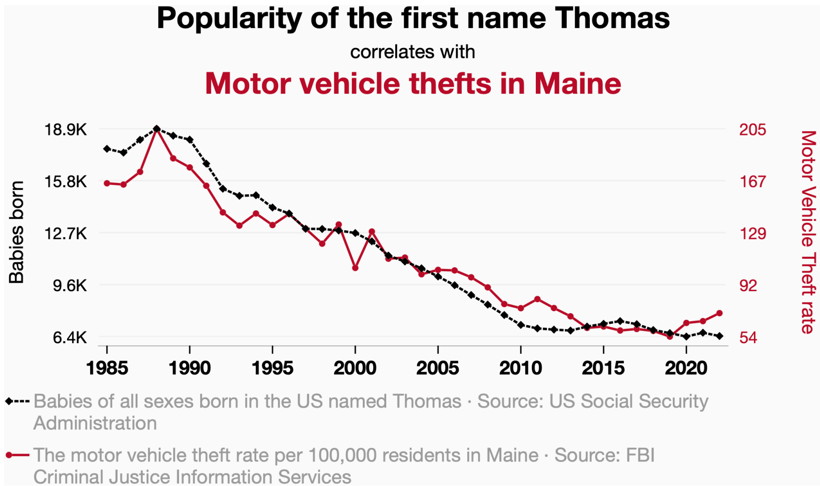
> "On the x-axis we have [x-variable] measured in [units] and on the y-axis the the [y-variable] measured in [units]. This figure shows the [change/pattern/relationship] between [x-variable] and [y-variable]. Overall [overall statement of pattern / trend / findings]."



Reading graphs practice

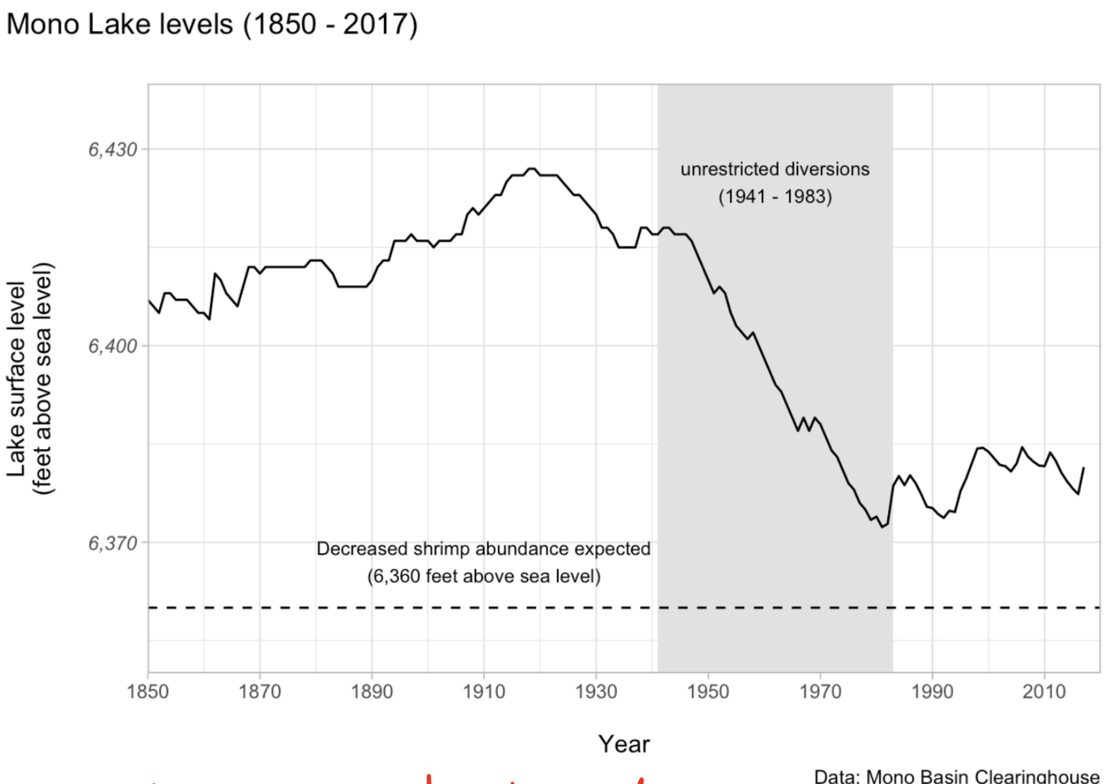
💬 Partner 2 gets to read the plot for the second variable, provide context for the data, and hypothesise an explanation.

> "On the x-axis we have [x-variable] measured in [units] and on the y-axis the the [y-variable] measured in [units]. This figure shows the [change/pattern/relationship] between [x-variable] and [y-variable]. Overall [overall statement of pattern / trend / findings]."



Source: TylerVigen.com

Reading graphs practice



* Find a second graph that students can practice.

Let’s take a 5 minute break

Back at

2:15 pm!

image: Flaticon.com

The exponential function

The number e

The **number e is a constant** in mathematics, it is approximately $e \approx 2.72$, but really its decimals continue infinitely without any pattern:

$$e = 2.718281828459045235 36...$$

It's exact value can be described as

$$e = 1 + 1 + \frac{1}{1*2} + \frac{1}{1*2*3} + \frac{1}{1*2*3*4} + \frac{1}{1*2*3*4*5} + \ldots$$

Like any other number, e follows normal power properties:

- $e^x \cdot e^y = \quad ? \quad = e^{x+y}$
- $\frac{1}{e^x} = \quad ? \quad = e^{-x}$
- $\frac{e^x}{e^y} = \quad ? \quad = e^{x-y}$
- $(e^x)^r = \quad ? \quad = e^{rx}$

✏️ Take a moment to write these with a single exponent.

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Like any other number, e follows normal power properties:

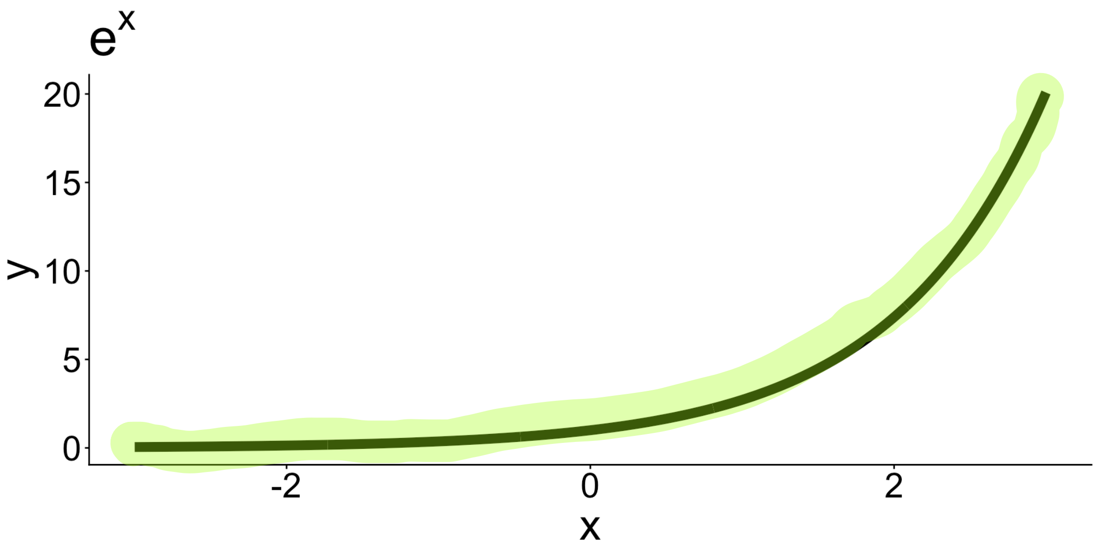
$$\begin{aligned} e^{x+y} &= e^x \cdot e^y \\ e^{-x} &= \frac{1}{e^x} \\ e^{x-y} &= \frac{e^x}{e^y} \\ e^{rx} &= (e^x)^r \end{aligned}$$

The exponential funciton

The **exponential function** e^x is the number e raised to the x power.

In function notation you may sometimes see it as

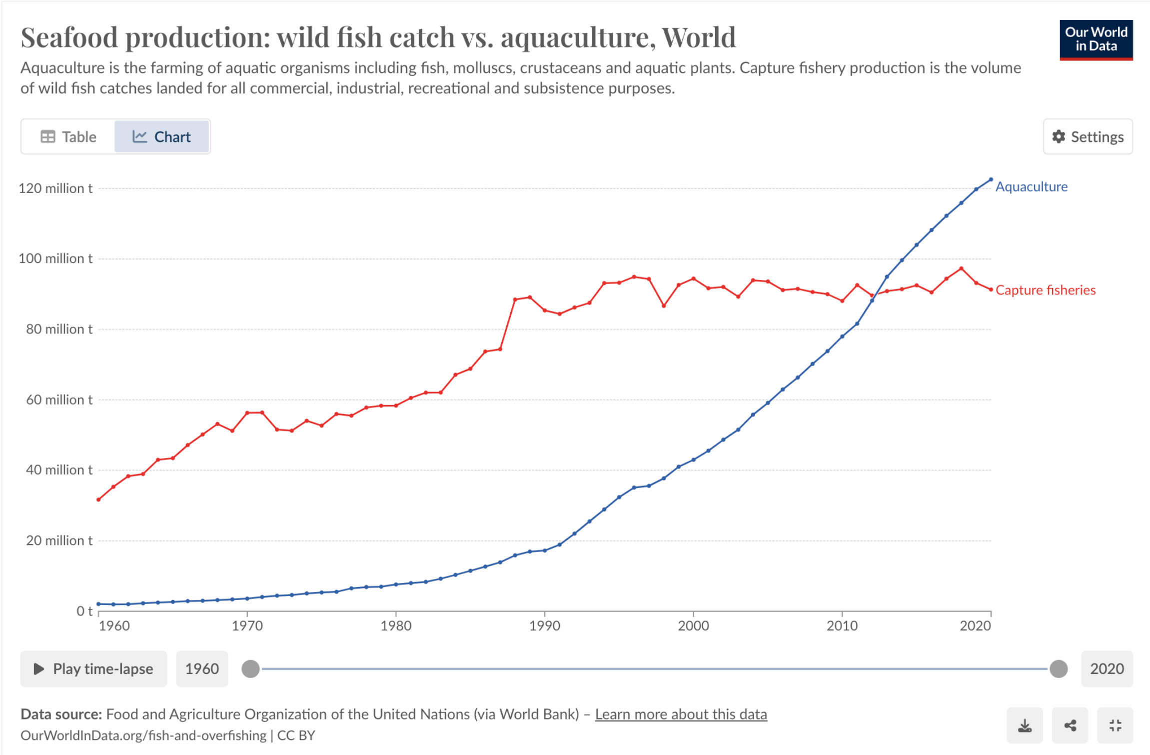
$$f(x) = e^x \quad \text{or} \quad \exp(x).$$



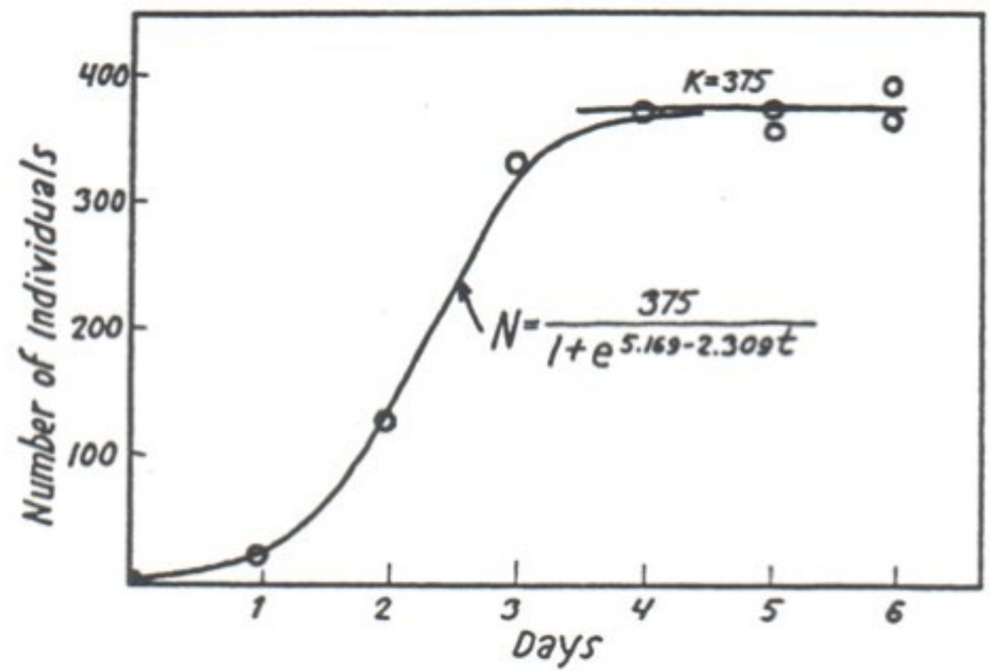
Why is the exponential function so common?

- **One reason:** Exponential trends show up a LOT in environmental science (the proportional change is the same over each time span)
- **Math reason:** Turns out it's a very useful value for calculus

A real-world example:



But populations can't grow exponentially forever



The growth of population of *Paramecium caudatum*

Gause, G. F. 1934. The Struggle for Existence. Baltimore: Williams and Wilkins.

Logistic growth

$\underline{\underline{e^0 = 1}}$

at $t = 0$

$$\frac{K}{1 + \frac{K - N_0}{N_0} \cdot e^{-r \cdot 0}}$$

$$\frac{\frac{a}{b}}{\frac{c}{d}} = \frac{ad}{bc}$$

$$N_t = \frac{K}{1 + [\frac{K - N_0}{N_0}]e^{-rt}}$$

$$= \frac{K}{1 + \frac{K - N_0}{N_0}}$$

$$\frac{K}{\frac{K}{N_0}} \qquad \frac{\frac{K}{1}}{\frac{K}{N_0}} = \frac{\cancel{K} N_0}{\cancel{K}} = N_0.$$

$$= \frac{K}{\frac{N_0}{N_0} + \frac{K - N_0}{N_0}}$$

Where N_t is the population size at time t, K is the carrying capacity, N_0 is the initial population size, and r is a growth rate.

$$= \frac{K}{\frac{\cancel{N_0} + K - \cancel{N_0}}{N_0}} = \frac{K}{\frac{K}{N_0}}$$

✏️ What should be the value of the formula when $t = 0$? Confirm your answer by substituting.

$$= \frac{N_0 K}{K}$$

We should always *understand the components of formulas and equations* - both conceptually and mathematically.

$$\underline{\underline{= N_0.}}$$

*add an additional slide to solve this exercise.

Logistic growth

$$N_t = \frac{K}{1 + [\frac{K - N_0}{N_0}]e^{-rt}}$$

1. Why might we expect logistic growth for many populations?
2. What variables *besides* time would influence the actual population?

Logarithms

Logarithms

$\log_a(b)$ is the exponent you need to raise a to get a value of b

Trying to answer a to the what gives b ?

For example:

- $\log_2(8) = x$ asks "to what power do I have to raise 2, to get a value of 8?"
- $\log_{105}(1) = x$ asks "to what power do I have to raise 105, to get a value of 1?"

2 to the what is 8 ?

$2^3 = 8$ this means $\log_2(8) = 3$.

$105^0 = 1$ this means $\log_{105}(1) = 0$

Natural logarithms

If $y > 0$ then $\ln(x)$ we read as natural logarithm of x

is the exponent you need to raise e to in order to get y.

$\ln(x)$ is asking e to the what equals x?

This means:

$$\left.\begin{aligned} \ln(e^{x}) &= x \\ e^{\ln(y)} &= y. \end{aligned}\right\}$$

$\ln(x)$ is the inverse of the exponential function

Natural logarithms

If $y > 0$, then $\ln(x)$ **is the exponent you need to raise e to in order to get y.**

This means:

$$\ln(e^x) = x$$
$$e^{\ln(y)} = y$$

In other words, $\ln(x)$ is the inverse of the exponential function e^x.

✏️ **Examples**

- $\ln(e^3)$ $= 3$
- $\ln(\frac{1}{e})$ $= \ln(e^{-1}) = -1.$
- $\ln(1)$ $= 0$: to what exponent should raise e to to get 1?
- $\ln(0)$ = does not exist: e to the what gives me 0?
- $\ln(-7)$ = undefined.

Natural logarithms

If $y > 0$, then $\ln(x)$ **is the exponent you need to raise e to in order to get y.**

This means:

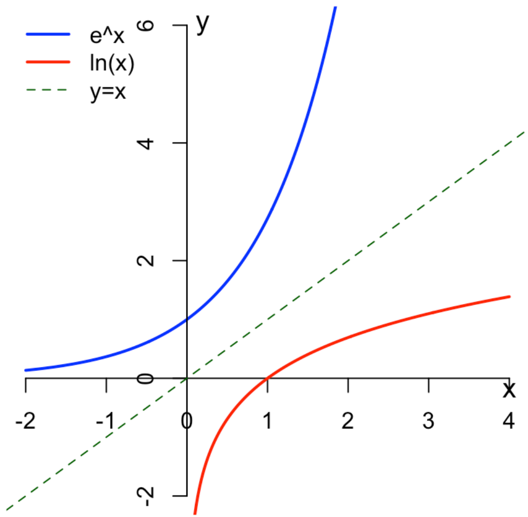
$$\ln(e^x) = x$$
$$e^{\ln(y)} = y$$

In other words, $\ln(x)$ is the inverse of the exponential function e^x.

Examples

- $\ln(e^3) = 3$ because 3 is the power of e needed to get e^3.
- $\ln(\frac{1}{e}) = \ln(e^{-1}) = -1$
- $\ln(1) = 0$ because 0 is the exponent we need to raise e to to get 1
- $\ln(0)$ does not exist, because there's no number we can raise e to to get 0
- $\ln(-7)$ does not exist, because e to any number always be positive

Natural log properties



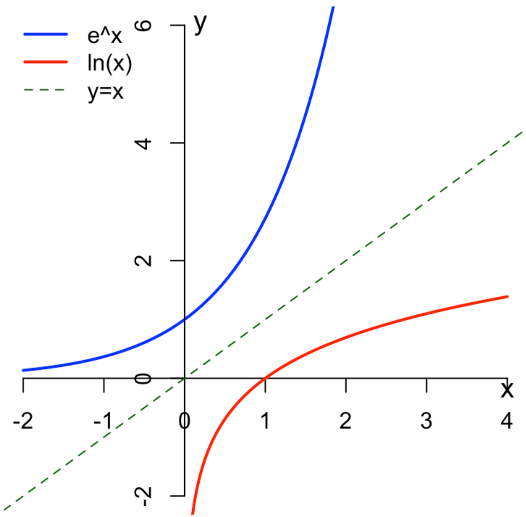
$$\ln(e^x) = x$$

$$\ln(xy) = \ln(x) + \ln(y)$$

$$\ln\left(\frac{x}{y}\right) = \ln(x) - \ln(y)$$

$$\ln(x^y) = y\ln(x).$$

Natural log properties



Properties

$$\ln(e^x) = x$$

$$\ln(xy) = \ln x + \ln y$$

$$\ln(\frac{x}{y}) = \ln x - \ln y$$

$$\ln(x^y) = y \ln x$$

Linear functions

Linear functions: slope-intercept form

✏️

Easiest model to describe linear relationships between two variables.

$$\underbrace{y}_{\substack{\text{output} \\ \text{dependent variable}}} = \overbrace{m}^{\text{slope}} \underbrace{x}_{\substack{\text{input} \\ \text{independent variable}}} + \underbrace{b}_{y \text{ intercept}}$$

$x = 0$ here:

$y = m \cdot 0 + b = b$

$(0, b)$ is on the line.

Linear functions: slope-intercept form

Easiest model to describe linear relationship between two variables!

$$\underbrace{y}_{\text{output}} = \overbrace{m}^{\text{Slope}} \underbrace{x}_{\text{input}} + \overbrace{b}^{y \text{ intercept}}$$

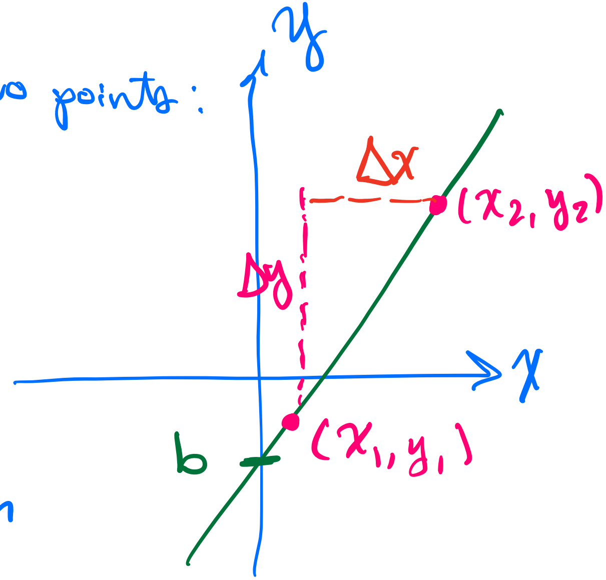
Finding the equation of a line

Given two points (x_1, y_1) and (x_2, y_2) on the line :

1. Find the slope m by using the two points:

$$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$

delta



2. Find the y-intercept b:
Since (x_1, y_1) is on the line, then

$$y_1 = m x_1 + b$$

Solve for b after plugging in the point:

$$b = y_1 - m x_1$$

Finding the equation of a line

Given two points (x_1, y_1) and (x_2, y_2) on the line:

1. Find m, the **slope**, by using the two points:

$$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}.$$

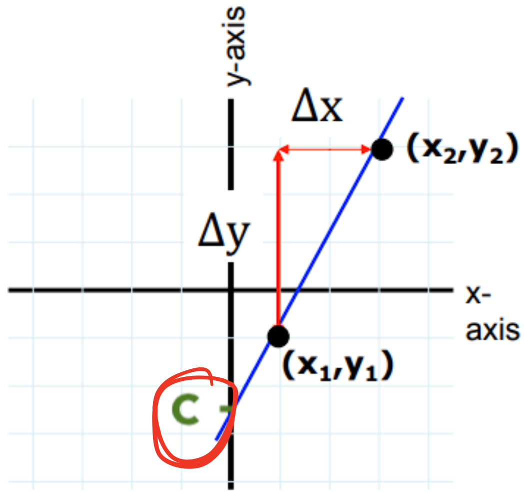
2. Find c, the **y-intercept**:

Since (x_1, y_1) is on the line, it satisfies:

$$y_1 = mx_1 + c.$$

We can solve for c to obtain:

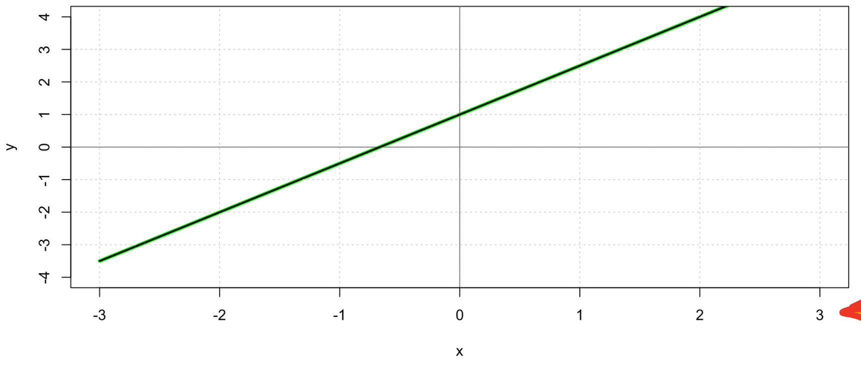
$$c = y_1 - mx_1.$$



* Update to be b instead of c to match previous explanation.

Equation of a line: practice

- Find the equation of the line that:
  a. passes through $(3,5)$ and $(2,-1)$.
  b. passes through $(2,-5)$ and has x-intercept equal to 4.
  c. has the following graph:



make number larger

- Make a graph showing the line that passes through (3,-1) and has slope equal to 2.

Equation of a line: solutions

Find the equation of the line that passes through $(3, 5)$ and $(2, -1)$.

✏️ Let's see a solution.

$$\frac{5-(-1)}{3-2} = \frac{6}{1}$$

$y = 6x + b$

$5 = 6(3) + b$

$5 = 18 + b$

$-13 = b$

$y = 6x - 13$

Equation of a line: solutions

Find the equation of the line that passes through $(2, -5)$ and has x-intercept equal to 4.

✏️ Let's see a solution.

$(4, 0)$

$\frac{0-(-5)}{4-2} = \frac{5}{2} = m$

$y = \frac{5}{2}x + b$

$-5 = \frac{5}{2}\left(\frac{2}{1}\right) + b$

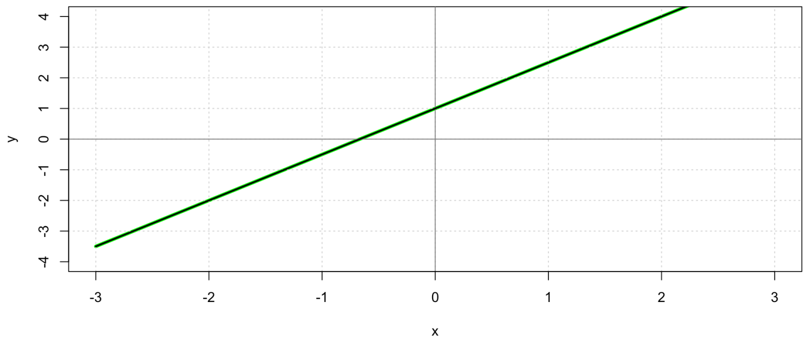
$-5 = 5 + b$

$-10 = b$

$y = \frac{5}{2}x - 10$

Equation of a line: solutions

Find the equation of the line that has the following graph:



$(-2, -2)$

$(0, 1)$

$$m = \frac{1 - -2}{0 - -2} = \frac{3}{2}$$

$$y = \frac{3}{2}x + 1$$

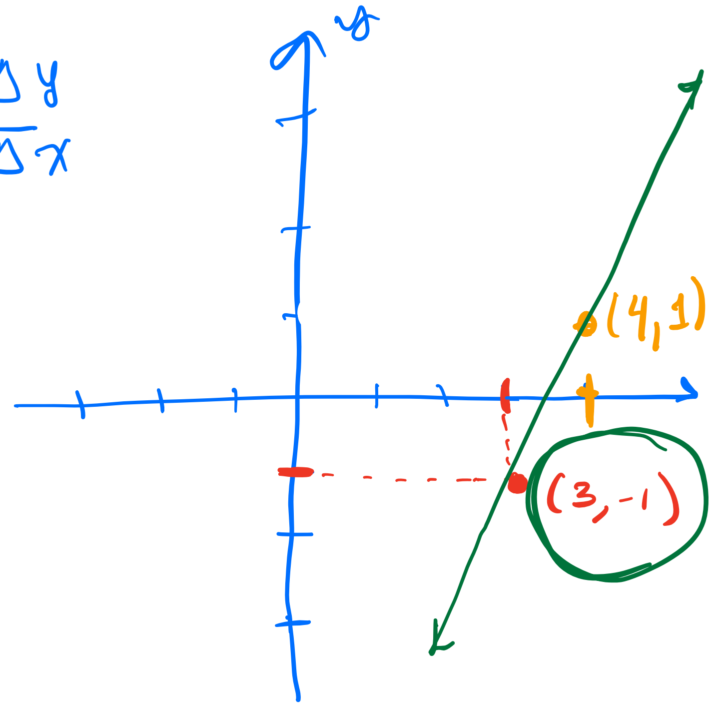
✏️ Let's see a solution.

Equation of a line: solutions

Make a graph showing the line that passes through $\overset{x}{3},\overset{y}{-1}$ and has slope equal to 2.

✏️ Let's see a solution.

$$\text{Slope} = 2 = \frac{2}{1} = \text{"}\frac{\text{rise}}{\text{run}}\text{"} = \frac{\Delta y}{\Delta x}$$



* Include solutions for these three exercises.

Interpreting the slope

Slope (average)

Sometimes, it can be useful to find the **average rate of change** of a function.

Between any two points (x_1, y_1) and (x_2, y_2) on the graph of a function, the slope is found by:

$$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$

= average rate of change between x_1 and x_2

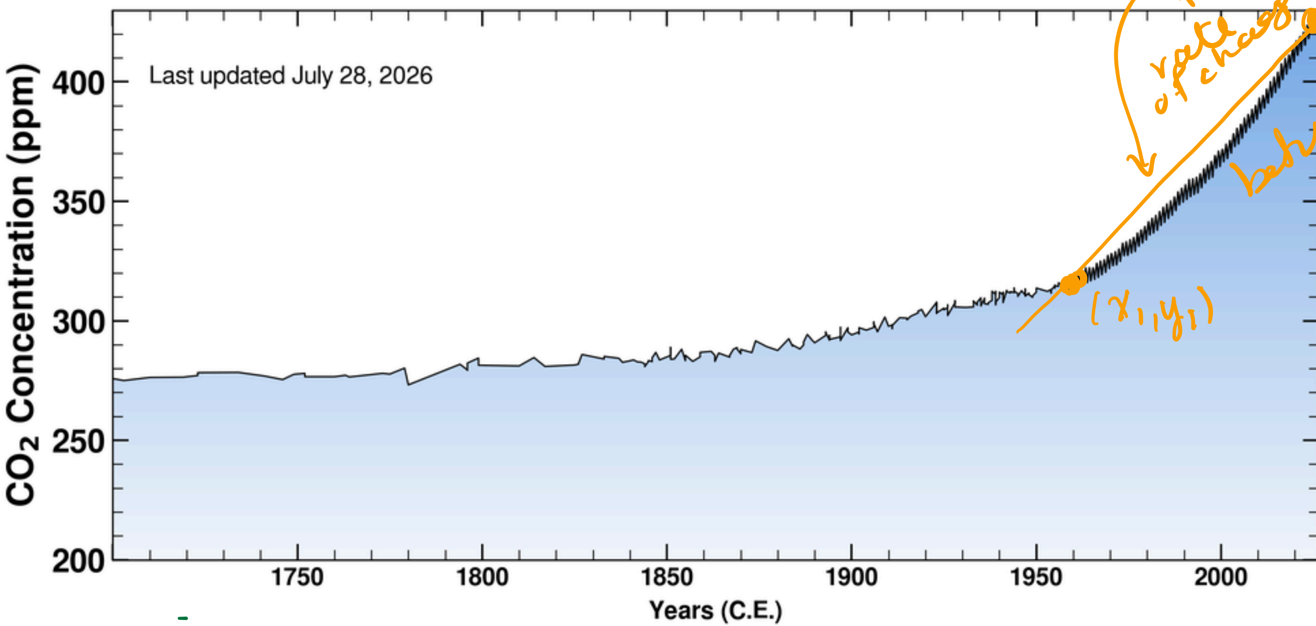
Same idea as in a line!

Slope represents rate of change

Ice-core data before 1958. Mauna Loa data after 1958.

slope of this line is the average rate of change between 1960 and 2010.

(x_2, y_2)



Source:The Keeling Curve

Get into the practice of saying the meaning out loud

- As if you're explaining it to someone unfamiliar with the data
- Including units
- Without overstating certainty

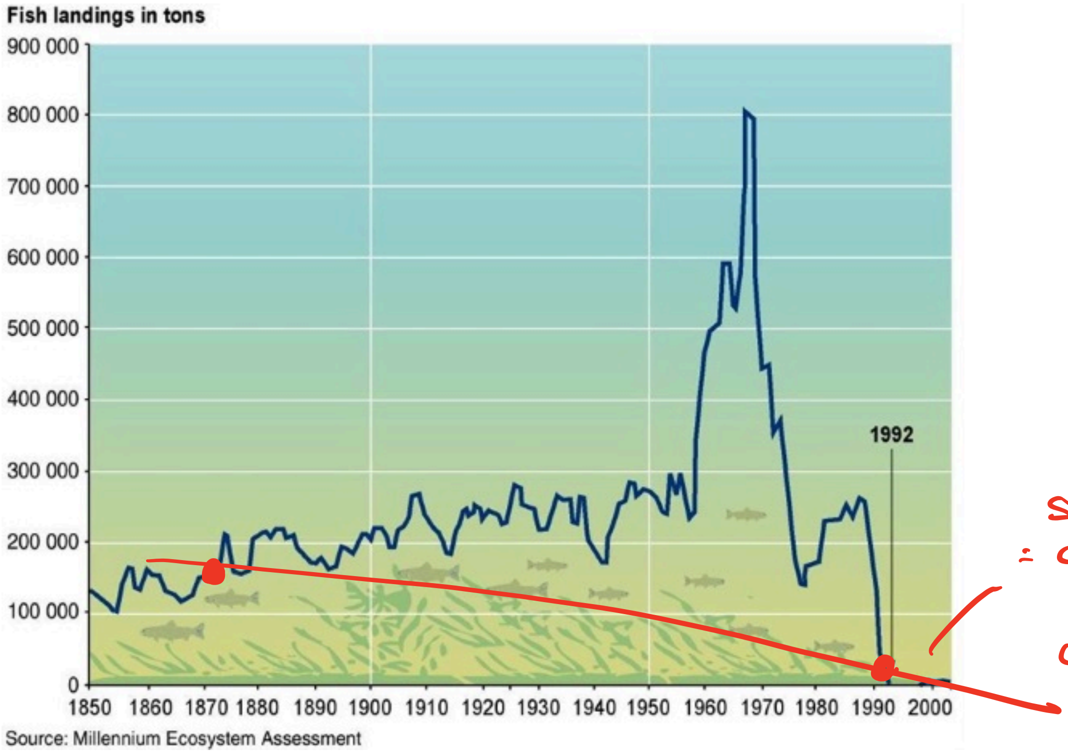
For example:

> "Between 1972 and 2020 the price of hobbit homes increased by an average of $2,450 per year"

differs from

> "Between 1972 and 2020 the price of hobbit homes increased by $2,450 per year."

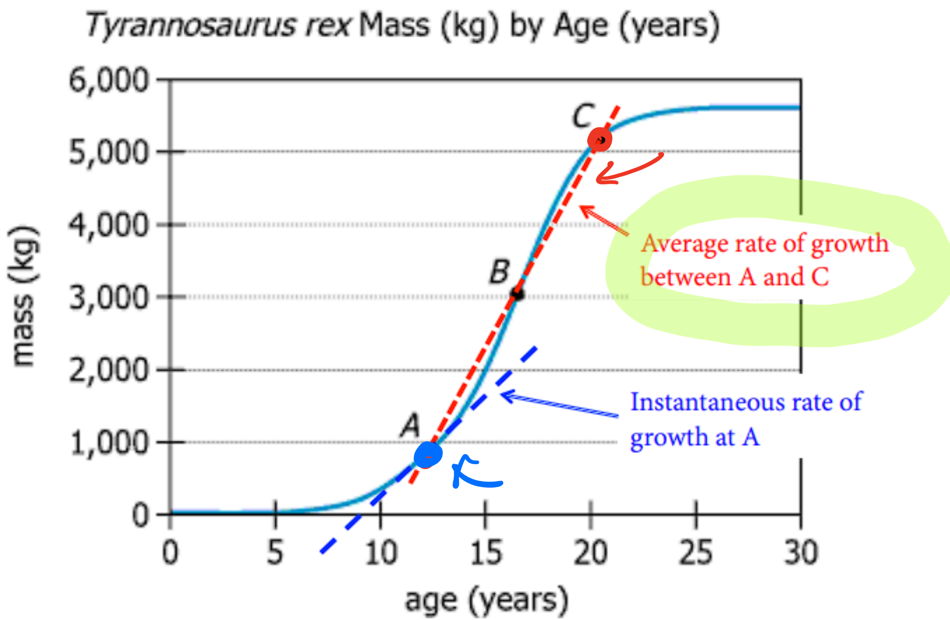
The *average* slope of a continuous function rarely tells the whole story



slope
= avg. rate
of
change

Slope can be *average* or *instantaneous*

- Rise over run can always be used to find average rate of change between two parts of a graph
- Instantaneous rate of change leads us to ✨calculus✨.



End of day 1!

What we covered today

- 🔢 Algebra review
- 🗺️ Solving equations
- ✖️ Exponents
- 📐 Polynomials
- 🌡️ Units
- 🧩 Definition of functions
- 📊 Graphing
- 💬 Reading graphs
- 📈 Linear functions
- ↗️ Slope as a rate of change

Update to include:

- exp
- ln